

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 1	Daily Tutorial Sheet 4
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46.(B) For the roots to be real, $D \geq 0$

$$\Rightarrow (-4)^2 - 4(1)(-\log_3 a) \geq 0 \Rightarrow \log_3 a \geq -4 \Rightarrow a \geq \frac{1}{81}$$

47.(A) $ax^2 + bx + c = 0$ For roots to be the reciprocal of the above equation, replace 'x' by $\frac{1}{x}$

$$\Rightarrow a\left(\frac{1}{x}\right)^2 + b\left(\frac{1}{x}\right) + c = 0 \Rightarrow cx^2 + bx + a = 0$$

48.(B) $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

Cross multiply to get : $x^2 + (a+b-2c)x + (ab-bc-ca) = 0$

Now, Roots of this equation are α and $-\alpha \Rightarrow$ Sum of roots = $\alpha + (-\alpha) = 0 = a + b - 2c$

$$\Rightarrow a + b = 2c$$

and Product of roots = $\alpha(-\alpha) = ab - bc - ca = ab - c(a+b) = ab - c(2c) [\because a+b=2c]$

$$= ab - 2c^2 = ab - 2\left(\frac{a+b}{2}\right)^2 [\because a+b=2c] = \frac{-1}{2}(a^2 + b^2)$$

49.(D) $8x^2 - 6x - a - 3 = 0$

Roots are $\alpha, \alpha^2 \Rightarrow \alpha + \alpha^2 = \frac{6}{8} = \frac{3}{4} \Rightarrow 4\alpha^2 + 4\alpha - 3 = 0$ or $\alpha = \frac{-3}{2}, \frac{1}{2}$ and $\alpha \cdot (\alpha^2) = \frac{-(a+3)}{8} \dots (i)$

$\alpha = \frac{-3}{2} \Rightarrow a = 24; \quad \alpha = \frac{1}{2} \Rightarrow a = -4$ [Use (i)]

50.(A) $x^2 - 6x + 5 \leq 0 \Rightarrow (x-5)(x-1) \leq 0$



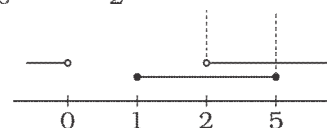
$x^2 - 2x > 0 \Rightarrow x(x-2) > 0$



Combine the two solutions to get :

$$\Rightarrow x \in [2, 5]$$

$\Rightarrow$ Integral values = 3, 4, 5 (3 values possible)



51.(B) $\frac{\alpha^3 + \beta^3}{\alpha^{-3} + \beta^{-3}} = \frac{\alpha^3 + \beta^3}{\frac{1}{\alpha^3} + \frac{1}{\beta^3}} = \frac{\alpha^3 + \beta^3}{\frac{\beta^3 + \alpha^3}{\alpha^3 \beta^3}} = (\alpha\beta)^3 = \frac{c^3}{a^3}$

52.(D) α, β are the roots of $ax^2 + bx + c = 0 \Rightarrow \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

$$\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)^2 = \left[\frac{\beta^2 - \alpha^2}{\alpha^2 \beta^2}\right]^2 = \frac{(\alpha + \beta)^2 (\alpha - \beta)^2}{(\alpha\beta)^4} = \frac{(\alpha + \beta)^2 [(\alpha + \beta)^2 - 4\alpha\beta]}{(\alpha\beta)^4} = \frac{(-b/a)^2 [(-b/a)^2 - 4(c/a)]}{(c/a)^4}$$

$$= \frac{b^2 a^2}{c^4} \left[\frac{b^2}{a^2} - \frac{4c}{a} \right] = \frac{b^2 (b^2 - 4ac)}{c^4}$$

53.(B) $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

Required equation is $x^2 - (\alpha + \beta + \alpha\beta)x + (\alpha + \beta)\alpha\beta = 0$

$$\Rightarrow x^2 - \left(\frac{-b}{a} + \frac{c}{a} \right)x + \left(\frac{-b}{a} \right) \left(\frac{c}{a} \right) = 0 \Rightarrow a^2 x^2 + a(b-c)x - bc = 0$$

54.(A) Let the roots of the equation be $m\alpha, n\alpha$. $\therefore m\alpha + n\alpha = -\frac{b}{a} \Rightarrow \alpha(m+n) = -\frac{b}{a} \Rightarrow \alpha = \frac{-b}{a(m+n)}$

$$m\alpha \cdot n\alpha = \frac{c}{a} \Rightarrow mn\alpha^2 = \frac{c}{a} \Rightarrow mn \frac{b^2}{a^2(m+n)^2} = \frac{c}{a} \Rightarrow mnb^2 = ac(m+n)^2$$

55.(A) $4x^2 + 2x + 1 = 4 \left(x^2 + \frac{1}{2}x + \frac{1}{4} \right) = 4 \left(x^2 + \frac{1}{2}x + \frac{1}{16} + \frac{1}{4} - \frac{1}{16} \right) = 4 \left(x + \frac{1}{4} \right)^2 + \frac{3}{4} \geq \frac{3}{4} \Rightarrow \frac{1}{4x^2 + 2x + 1} \leq \frac{4}{3}$

56.(B) Here, $f(x) = 4x^2 - 2(1+2+3+4)x + 30 = 4x^2 - 20x + 30 = 4(x^2 - 5x) + 30$

$$= 4 \left\{ x^2 - 5x + \frac{25}{4} \right\} + 30 - 25 = 4 \left(x - \frac{5}{2} \right)^2 + 5$$

Now, $f(x)$ is minimum. When $\left(x - \frac{5}{2} \right)^2$ is minimum, i.e., when $\left(x - \frac{5}{2} \right)^2 = 0$, i.e., when $x = \frac{5}{2}$.

57.(C) $\left| \frac{x^2 - 1}{x^2 + x + 1} \right| < 1 \Rightarrow -1 < \frac{x^2 - 1}{x^2 + x + 1} < 1$

Case I: $\frac{x^2 - 1}{x^2 + x + 1} > -1 \Rightarrow \frac{x^2 - 1 + x^2 + x + 1}{x^2 + x + 1} > 0$

$$\Rightarrow 2x^2 + x > 0 \left[\because x^2 + x + 1 > 0 \forall x \in R \text{ as } a > 0 \text{ and } D < 0 \right] \Rightarrow x(2x + 1) > 0$$

$$\Rightarrow x(2x + 1) > 0 \Rightarrow x \in \left(-\infty, -\frac{1}{2} \right) \cup (0, \infty)$$

Case-II: $\frac{x^2 - 1}{x^2 + x + 1} < 1 \Rightarrow \frac{x^2 - 1 - x^2 - x - 1}{x^2 + x + 1} < 0$

$$\Rightarrow x + 2 > 0$$

Therefore $x \in \left(-2, -\frac{1}{2} \right) \cup (0, \infty)$

Alternate Solution:

$$\left| \frac{x^2 - 1}{x^2 + x + 1} \right| = \frac{|x^2 - 1|}{|x^2 + x + 1|} < 1$$

$$\Rightarrow |x^2 - 1| < |x^2 + x + 1|$$

{Since $|x^2 + x + 1| > 0$, we can easily cross multiply without changing the inequality}

$$\Rightarrow |x^2 - 1| < x^2 + x + 1 \quad \{x^2 + x + 1 > 0 \quad \forall x \in R\}$$

2 cases arise

$$\text{Case-I: } x^2 - 1 \geq 0 \Rightarrow x \in (-\infty, -1] \cup [1, \infty) \quad \dots (i)$$

$$\Rightarrow |x^2 - 1| = x^2 - 1 < x^2 + x + 1$$

$$\Rightarrow x > -2 \quad \dots (ii)$$

$$(i) \cap (ii) \Rightarrow x \in (-2, -1] \cup [1, \infty) \quad \dots (iii)$$

$$\text{Case-II: } x^2 - 1 < 0 \Rightarrow x \in (-1, 1) \quad \dots (iv)$$

$$\Rightarrow |x^2 - 1| = 1 - x^2 < 1 + x + x^2 \Rightarrow 2x^2 + x > 0$$

$$\Rightarrow x \in (-\infty, -\frac{1}{2}) \cup (0, \infty) \quad \dots (v)$$

$$(iv) \cap (v) \Rightarrow x \in (-1, -\frac{1}{2}) \cup (0, 1) \quad \dots (vi)$$

$$(iii) \cup (vi) \Rightarrow x \in (-2, -\frac{1}{2}) \cup (0, \infty)$$

58.(D) Given equation is $x^2 + x - |x| - 4 = 0 \quad \dots (i)$

Two cases arise:

Case 1: If $x \geq 0$, then $|x| = x$ and then the equation becomes $x^2 + x - x - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
 $x = 2$ is a solution in this case.

Case 2: If $x < 0$, then $|x| = -x$ and then equation becomes $x^2 + x - (-x) - 4 = 0 \Rightarrow x^2 + 2x - 4 = 0$

$$\Rightarrow x = \frac{-2 \pm \sqrt{4 + 16}}{2} \Rightarrow x = -1 \pm \sqrt{5}, \text{ but } x < 0, \text{ therefore, } x = -1 - \sqrt{5}$$

59.(A) Consider $x^2 + 4x + 3 = (x+1)(x+3) \Rightarrow x^2 + 4x + 3 \geq 0$ for $x \in (-\infty, -3] \cup [-1, \infty)$ and $x^2 + 4x + 3 < 0$ for $x \in (-3, -1)$

Let us take $x \in (-\infty, -3] \cup [-1, \infty)$. Then $|x^2 + 4x + 3| + (2x + 5) = 0$ gives $(x^2 + 4x + 3) + (2x + 5) = 0$

$$\text{or } x^2 + 6x + 8 = 0 \Rightarrow x = \frac{-6 \pm 2}{2} = -2, -4$$

$$\text{Reject } x = -2 \quad \text{So, } x = -4.$$

Now take $x \in (-3, -1)$

Then $|x^2 + 4x + 3| + (2x + 5) = 0$ gives $-(x^2 + 4x + 3) + 2x + 5 = 0$

$$\text{or } -x^2 - 2x + 2 = 0 \quad \text{or } x^2 + 2x - 2 = 0 \quad \text{or } x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$$

$$\text{Reject } x = -1 + \sqrt{3} \quad \text{So, } x = -1 - \sqrt{3}$$

60.(D) Case 1: $x \geq 1$ $2x + 3 - x + 1 = 6 \Rightarrow x = 2$

Case 2: $-\frac{3}{2} \leq x \leq 1$ $2x + 3 + x - 1 = 6 \Rightarrow x = \frac{4}{3}$ (Not possible)

Case 3: $x \leq -\frac{3}{2}$ $-2x - 3 + x - 1 = 6 \Rightarrow x = -10$ $\therefore x \in \{-10, 2\}$

Hence, sum of roots = -8